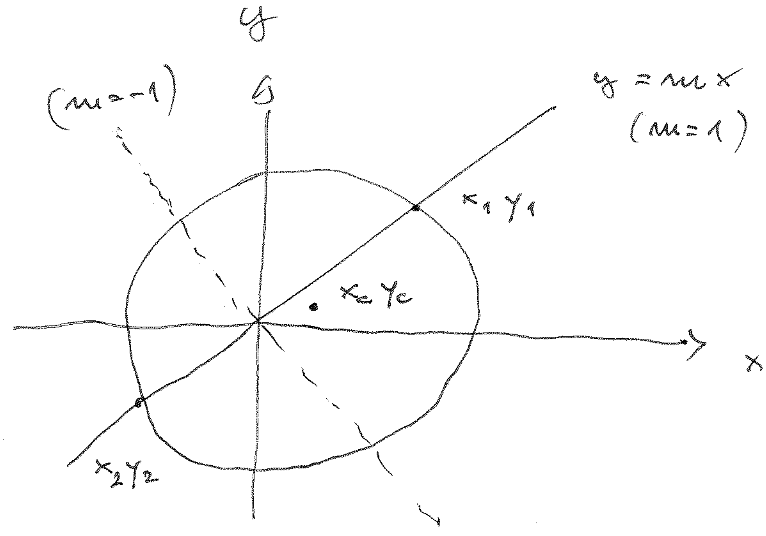


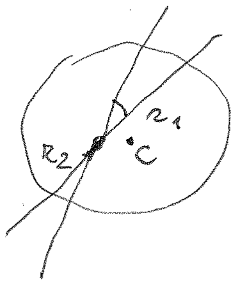
$x_c^2+y_c^2<r^2$

$x_c \geq y_c$

equaz. circf

$(x-x_c)^2+(y-y_c)^2=r^2$

Area spicchio (infinitesimale)



$$(r_1^2+r_2^2)\,d\theta = (x_1^2+y_1^2+x_2^2+y_2^2)\,d\theta =$$

$$A = (x_1^2+x_2^2)(1+m^2)\,d\theta$$

$$\operatorname{tg}\theta = m \;;\quad \frac{dm}{d\theta} = (1+\operatorname{tg}^2\theta) = (1+m^2) \;;\quad d\theta = \frac{dm}{(1+m^2)}$$

$$A=(x_1^2+x_2^2)\,dm$$

Esprimiamo x_1 x_2 e $x_1^2+x_2^2$ in funzione di m (intersezioni retta circonferenza; visto che l'origine è INTERNA alla circonferenza, ci sono sempre 2 intersezioni)

$$(x-x_c)^2+(mx-y_c)^2=r^2$$

$$\underset{(a)}{x^2(1+m^2)} - \underset{(b)}{2x(x_c+my_c)} + \underset{(c)}{x_c^2+y_c^2-r^2} = \varnothing$$

$$x_1^2+x_2^2 = S^2-2P = \frac{b^2}{a^2}-2\frac{c}{a}$$

$$x_1^2+x_2^2 = \frac{4(x_c+my_c)^2}{(1+m^2)^2} + 2\,\frac{(r^2-x_c^2-y_c^2)}{(1+m^2)} =$$

$$= \frac{2}{1+m^2}\cdot\frac{2x_c^2+2m^2y_c^2+4x_cy_cm+r^2(1+m^2)-x_c^2(1+m^2)-y_c^2(1+m^2)}{1+m^2}$$

$$= \frac{2}{1+m^2} \quad \frac{x_c^2(1-m^2) + y_c^2(1-m^2) + 4x_c y_c m + 2^2(1+m^2)}{1+m^2}$$

Ora si tratta di INTEGRARE in dm con gli estremi opportuni
A numeratore c'è un polinomio di 2° grado, a denominatore di 4°

$$= \frac{2}{(1+m^2)^2} \cdot \left[m^2(2^2 - x_c^2 - y_c^2) + 4x_c y_c m + x_c^2 + y_c^2 + 2^2 \right]$$

Vedo i 3 termini separatamente

$$\int \frac{K_1 m}{(1+m^2)^2} dm \qquad 1+m^2 = n \qquad \frac{dn}{dm} = 2m$$

$$2m\, dm = dn$$

$$\frac{K_1}{2} \int \frac{dn}{n^2} = -\frac{K_1}{2} n^{-1} + \text{cost} = -\frac{K_1}{2} \frac{1}{(m^2+1)} + \text{cost}$$

$$\int \frac{K_2}{(1+m^2)^2} dm \qquad m = tg\theta$$

$$dm = d\theta\,(1+tg^2\theta)$$

$$K_2 \int \frac{1}{(1+tg^2\theta)^2} d\theta\,(1+tg^2\theta) = K_2 \int \frac{1}{1+tg^2\theta} d\theta = K_2 \int \cos^2\theta\, d\theta = \quad \frac{1+\cos 2\theta}{2}$$

$$= \frac{K_2}{4} \int (1+\cos 2\theta)\, d2\theta = \frac{K_2}{2}\theta + \frac{K_2 \sin 2\theta}{4} + \text{cost}$$

$$\int \frac{K_3 m^2}{(1+m^2)^2} dm = K_3 \int \frac{tg^2\theta}{(1+tg^2\theta)} d\theta = K_3 \int \sin^2\theta\, d\theta \quad \frac{1-\cos 2\theta}{2}$$

$$= \frac{K_3}{4} \int (1-\cos 2\theta)\, d2\theta = \frac{K_3}{2} \cdot \theta - \frac{K_3 \sin 2\theta}{4} + \text{cost}$$

Ora che abbiamo le primitive, valutiamo gli integrali sommando gli intervalli $0 \leq m \leq 1 \quad (0 \leq \theta \leq \pi/4)$

$-\infty < m \leq -1 \quad (\pi/2 \leq \theta \leq 3\pi/4)$

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contributo 1 $\quad K_1 = 8 x_c y_c$

$$8 x_c y_c \left[\left(-\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \right) + \left(-\frac{1}{2} \cdot \frac{1}{2} + 0 \right) \right] = \varnothing \quad !$$

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contributo 2 $\quad K_2 = 2 (x_c^2 + y_c^2 + r^2)$

$$(x_c^2 + y_c^2 + r^2) \left[\left(\frac{\pi}{4} + \frac{1}{2} \right) + \left(\frac{\pi}{4} - \frac{1}{2} \right) \right]$$

contributo 3 $\quad K_3 = 2 (r^2 - x_c^2 - y_c^2)$

$$(r^2 - x_c^2 - y_c^2) \left[\left(\frac{\pi}{4} - \frac{1}{2} \right) + \left(\frac{\pi}{4} + \frac{1}{2} \right) \right]$$

Sommando i 3 contributi (spicchi 1, 3, 5 e 7) si ha

$A_{tot} = \frac{\pi}{2} \cdot r^2$ cioè esattamente metà del totale!